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Research on Option Pricing Method Based on the Black-Scholes Model

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Abstract: Option pricing is one of the core research topics in the field of financial engineering. As a classical option pricing method, the Black-Scholes model provides a significant foundation for both theory and practice in modern financial markets. This paper first elaborates on the theoretical foundation and mathematical derivation of the Black-Scholes model, analyzes its practical applications in option pricing, and explores its limitations, including the strict assumptions about market conditions and its applicability in environments with fluctuating volatility or sudden market jumps. To address these issues, this study improves the Black-Scholes model from two perspectives: stochastic volatility and jump-diffusion models. The improved models are validated through experimental designs, and their effectiveness is compared with the traditional model. The experimental results demonstrate that the improved models can provide more accurate pricing results in complex market environments. This research not only deepens the understanding of the Black-Scholes model but also offers new insights and approaches for pricing complex financial instruments.

Keywords: Black-Scholes model; option pricing; stochastic volatility; jump-diffusion; financial engineering

1. Introduction

Options, as key financial derivatives, play a critical role in modern financial markets by offering tools for risk hedging and portfolio optimization. The Black-Scholes model, a cornerstone of modern finance, provides a rigorous mathematical framework for option pricing based on the stochastic processes of asset prices, significantly advancing the growth of financial derivatives markets. However, the model's core assumptions—frictionless markets, geometric Brownian motion, and constant volatility—often diverge from real-world market conditions, resulting in pricing inaccuracies. To address these limitations, researchers have introduced improvements such as stochastic volatility models to capture volatility dynamics and jump-diffusion models to account for sudden price changes. While these enhancements improve the model's accuracy, challenges remain in balancing complexity and applicability, especially in complex markets. This study systematically reviews the Black-Scholes model, evaluates its limitations, and validates improvements through experiments, offering theoretical and practical insights into option pricing in dynamic financial markets [1].

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2. Theoretical Foundation of the Black-Scholes Model

The Black-Scholes model is one of the most influential option pricing methods in the field of financial engineering. Its theoretical framework is built on a rigorous mathematical foundation and a series of key assumptions. These assumptions ensure that the model's derivation can be simplified into a mathematically solvable form while providing a clear logical basis for option pricing. Figure 1 illustrates the core partial differential equation and key formula of the Black-Scholes model, which describe the dynamic behavior of option prices over time and changes in asset prices, laying the groundwork for subsequent theoretical derivations and practical applications [2].

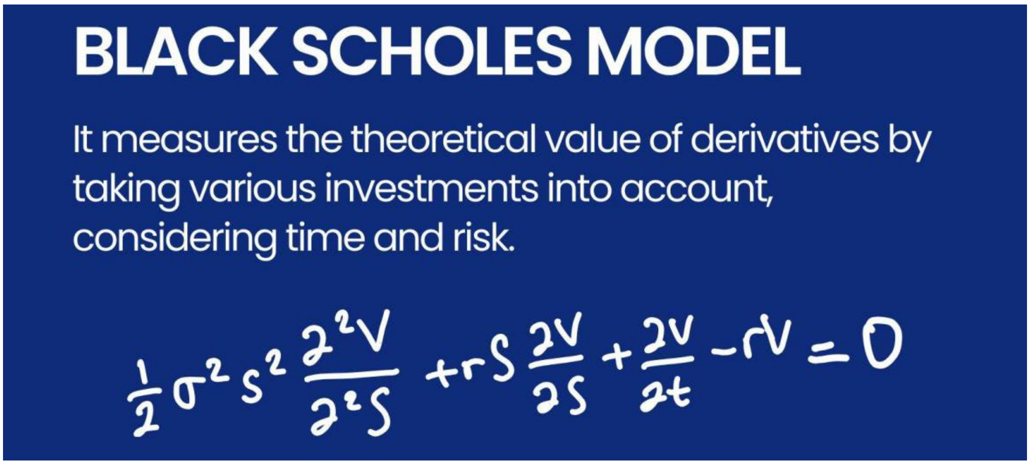

Figure 1. The Partial Differential Equation and Theoretical Implications of the Black-Scholes Model.

2.1. Basic Assumptions of the Black-Scholes Model

The Black-Scholes model is based on the following key assumptions, which not only render the model mathematically solvable but also reflect the idealized market conditions for option pricing: First, the market is assumed to be frictionless. This means there are no transaction costs, taxes, or other frictional factors, allowing investors to buy and sell assets without limitations [3]. Additionally, the market is fully liquid, enabling investors to trade at the current market price at any time, with asset prices entirely determined by supply and demand, free from manipulation or arbitrage opportunities. Second, the dynamic changes in the price of the underlying asset are assumed to follow a geometric Brownian motion, as illustrated in Figure 1. This assumption implies that the logarithmic returns of asset prices follow a normal distribution, with the price changes comprising two components: a drift term representing the long-term trend (μ) and a stochastic term describing short-term volatility (σ). The stochastic process is mathematically expressed in Formula 1:

$$dS = \mu S dt + \sigma S dW \quad (1)$$

Third, the Black-Scholes model assumes that volatility (σ) and the risk-free interest rate (r) are known and constant. While the assumption of constant volatility simplifies the theoretical derivation, in real markets, volatility often changes with time and market conditions. Furthermore, the model assumes that the underlying asset does not pay dividends or other forms of income during the option's validity period, a condition primarily applicable to non-dividend-paying stock options. Finally, the market is assumed to be arbitrage-free. Under this assumption, investors cannot achieve risk-free profits by combining spot and option assets. This assumption is directly related to market efficiency and serves as the foundation for constructing risk-free portfolios in the Black-Scholes model derivation. In summary, the basic assumptions of the Black-Scholes model provide mathematical feasibility for its theoretical derivation and a framework foundation for its practical application. However, these idealized characteristics also limit the model's applicability in complex market environments, paving the way for further research into model improvements [4].

2.2. Derivation Process and Mathematical Expression

The derivation of the Black-Scholes model is based on the principle of risk-free arbitrage. By constructing a risk-free portfolio composed of stocks and options, a partial differential equation describing the changes in option prices is derived, as shown in Figure 1. Additionally, Figure 2 visually demonstrates how option prices are affected by stock prices and strike prices, providing an intuitive understanding of the model's fundamental principles and pricing logic [5].

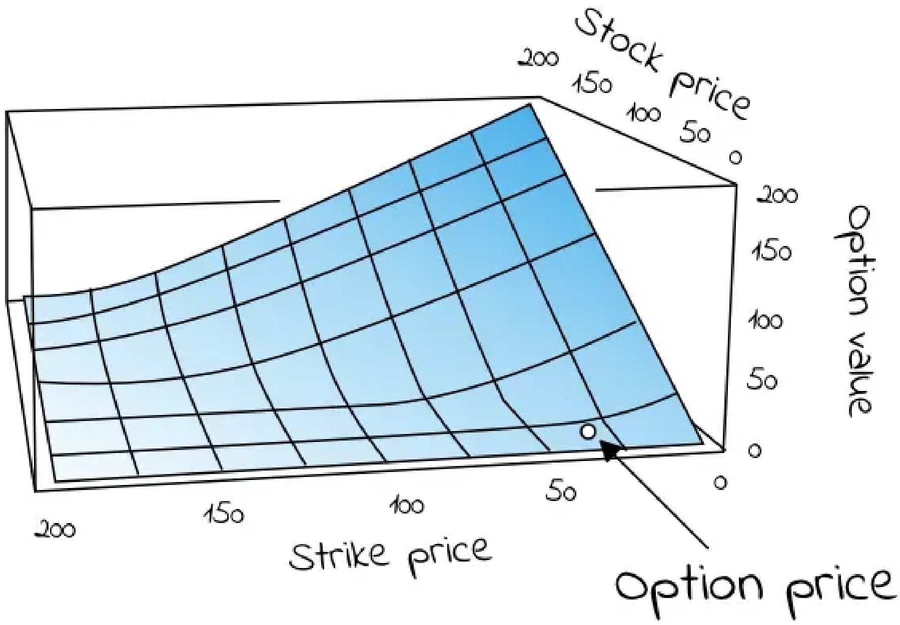

Figure 2. The Three-Dimensional Relationship Between Option Prices, Strike Prices, and Stock Prices Under the Black-Scholes Model.

The derivation begins by assuming a financial asset whose price SS follows a geometric Brownian motion as shown in Formula 2:

$$dS = \mu S dt + \sigma S dW \quad (2)$$

where μrepresents the expected return of the asset, $\sigma$is the volatility, and W is a standard Brownian motion. By applying Itô's Lemma, the change in the option price V(S,t) is derived. Next, a risk-free portfolio is constructed by combining options and the underlying asset, assuming the portfolio's return equals the risk-free rate r. The following partial differential equation is obtained as shown in Formula 3:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\sigma^2 V}{\partial S^2} - rV = 0 \quad (3)$$

This equation, known as the Black-Scholes partial differential equation, forms the core mathematical expression of the model. Solving this equation yields the explicit pricing formulas for European call and put options. For a European call option, the pricing is as shown in Formula 4:

$$C = S_0 N(d_1) - Xe^{-rT} N(d_2) \quad (4)$$

Here, S_0 is the current stock price, X is the strike price, T is the time to maturity, r is the risk-free rate, and N () is the cumulative distribution function of the standard normal distribution. Figure 2 vividly illustrates the dynamic changes in option prices under varying stock prices and strike prices. This visualization enhances the understanding of the model's flexibility and applicability in practical scenarios. Moreover, the mathematical framework laid by this model serves as a theoretical foundation for subsequent advancements, such as stochastic volatility models and jump-diffusion models. In conclusion, the Black-Scholes model, through rigorous mathematical derivation and logical assumptions, successfully constructs a framework that effectively reflects market conditions for option pricing. However, to better adapt to the complexities of real markets, further expansion and optimization of the model remain focal points for researchers [6].

3. Applications and Limitations of the Black-Scholes Model

The Black-Scholes model, as a critical tool in modern financial engineering, has been widely applied in real-world financial markets. However, the strict assumptions underlying the model also limit its applicability in certain complex market environments. This section explores the practical applications and limitations of the Black-Scholes model to provide a comprehensive analysis of its theoretical and practical value [7].

3.1. Practical Applications of the Model

The Black-Scholes model is most commonly used for pricing European options. Through its formula, investors can quickly calculate the theoretical price of options, assisting market participants in making trading decisions. Whether for stock options or index options, the Black-Scholes model demonstrates high computational efficiency and theoretical applicability. For instance, in the stock market, investors can use the Black-Scholes model to evaluate the fair price of a call option and determine whether a specific financial derivative is worth investing in. Moreover, financial institutions frequently employ the model as a foundational tool for constructing and managing complex portfolios. Figure 2 illustrates the relationship between option price, strike price, and stock price, providing intuitive guidance for developing option trading strategies. Specifically, when the stock price approaches the option's strike price, the option's value becomes highly sensitive to price changes. This sensitivity offers investors valuable insights for optimizing their trading timing. Additionally, the Black-Scholes model serves as the theoretical foundation for the innovation of financial derivatives, with many new types of options and financial derivatives designed based on the model's logical framework. In risk management, the Black-Scholes model is extensively used to formulate hedging strategies. By calculating option "Greeks" (e.g., Delta, Gamma), investors can dynamically adjust their portfolios to hedge against risks arising from market price fluctuations. For example, using the Delta value calculated by the model, investors can buy or sell the underlying asset to hedge price changes in their option positions, thereby achieving risk-free arbitrage. Furthermore, the Black-Scholes model is applied in various quantitative finance scenarios. For example, in corporate mergers and acquisitions, the model can be used to evaluate the value of real options, supporting strategic investment decisions. In credit risk analysis, the theoretical foundation of the Black-Scholes model helps explain the market pricing and default risk of corporate debt. This demonstrates that while the model was originally designed for option pricing, its mathematical logic and theoretical framework possess strong versatility. Despite the numerous advantages of the Black-Scholes model in practical applications, its performance heavily depends on whether market conditions align with its basic assumptions. For instance, when market volatility is relatively stable, and the underlying asset does not pay dividends, the model typically provides accurate pricing results. However, in situations where volatility fluctuates frequently, or market conditions are complex, the model may exhibit significant pricing errors. This aspect will be explored further in the following discussion on the model's limitations. In summary, the Black-Scholes model, as a crucial tool in financial markets, provides not only accurate theoretical pricing formulas but also robust decision-making support for investors and institutions. Although its basic assumptions limit its applicability in certain scenarios, its broad usability and significant contributions to the development of financial engineering are undeniable [8].

3.2. Analysis of the Model's Limitations

Although the Black-Scholes model has achieved significant success in option pricing and has been widely applied, its idealized assumptions lead to certain limitations in complex market environments. These limitations not only affect the model's accuracy in specific markets but also restrict its applicability to new financial instruments. Therefore, a comprehensive analysis of the model's limitations can deepen our understanding of its theoretical foundation and provide directions for improvement. First, the Black-Scholes model assumes a frictionless market, meaning no transaction costs, taxes, or constraints exist. However, in real markets, transaction costs are unavoidable, especially for high-frequency traders, where transaction costs can significantly impact the accuracy of option pricing. Additionally, market liquidity varies across assets, and in low-liquidity markets, asset prices may not fully reflect market information in a timely manner, affecting the model's precision. Second, the model assumes that asset prices follow a geometric Brownian motion, and volatility (σ) is known and constant. While this assumption simplifies mathematical derivation, it overlooks the dynamic nature of volatility and phenomena like the "volatility smile." In reality, volatility often changes with market sentiment and economic conditions. For instance, during financial crises or major events, market volatility can fluctuate dramatically, a dynamic characteristic not captured by the Black-Scholes model, leading to significant pricing errors in such scenarios. Third, the model assumes a constant risk-free interest rate (r), whereas real-world interest rates are influenced by macroeconomic policies and supply-demand dynamics, causing them to fluctuate over time. Moreover, the model's assumption that the underlying asset does not pay dividends before option expiration is also limited in practical applications [9]. For instance, ignoring dividend payments for high-dividend-paying stock options may cause the model's results to deviate from actual prices. While adjustments to the model can address this issue partially, they add complexity to the calculations. Additionally, the Black-Scholes model assumes no arbitrage opportunities exist and that investors can construct perfectly hedged risk-free portfolios. However, in real markets, constructing a perfectly hedged portfolio is often constrained by transaction costs, liquidity, and market conditions, reducing the model's reliability in arbitrage pricing. For instance, during periods of significant market volatility or sudden price jumps, hedging strategies may not fully cover price change risks, rendering the model ineffective. Finally, the Black-Scholes model also exhibits limitations in handling complex derivatives (e.g., American options, barrier options). The characteristics of these derivatives require the model to handle more boundary conditions and path dependencies, where the traditional Black-Scholes model falls short. Pricing these financial instruments necessitates the introduction of more sophisticated models or numerical methods, such as Monte Carlo simulations or finite difference methods. In conclusion, while the Black-Scholes model has laid a critical foundation for option pricing theory and practice, its limitations become increasingly apparent in complex market environments. These limitations primarily stem from the discrepancies between the model's assumptions and real-world market conditions, including dynamic volatility, market frictions, and the adaptability to complex financial instruments. These challenges provide rich directions for further research into model improvements and extensions, such as stochastic volatility models, jump-diffusion models, and the incorporation of numerical methods. These advancements not only address the shortcomings of the Black-Scholes model but also contribute new ideas to the further development of financial engineering.

4. Methods to Improve the Black-Scholes Model

The Black-Scholes model provides a simple and effective mathematical framework for option pricing, but its strict assumptions limit its applicability in complex market environments. To address this issue, the jump-diffusion model has been proposed as an improvement, better capturing the dramatic changes and discontinuities in asset prices. Figure 3 vividly illustrates the three-dimensional relationship between option price, stock price, and time under the jump-diffusion model, offering a visual representation of this improved approach [10].

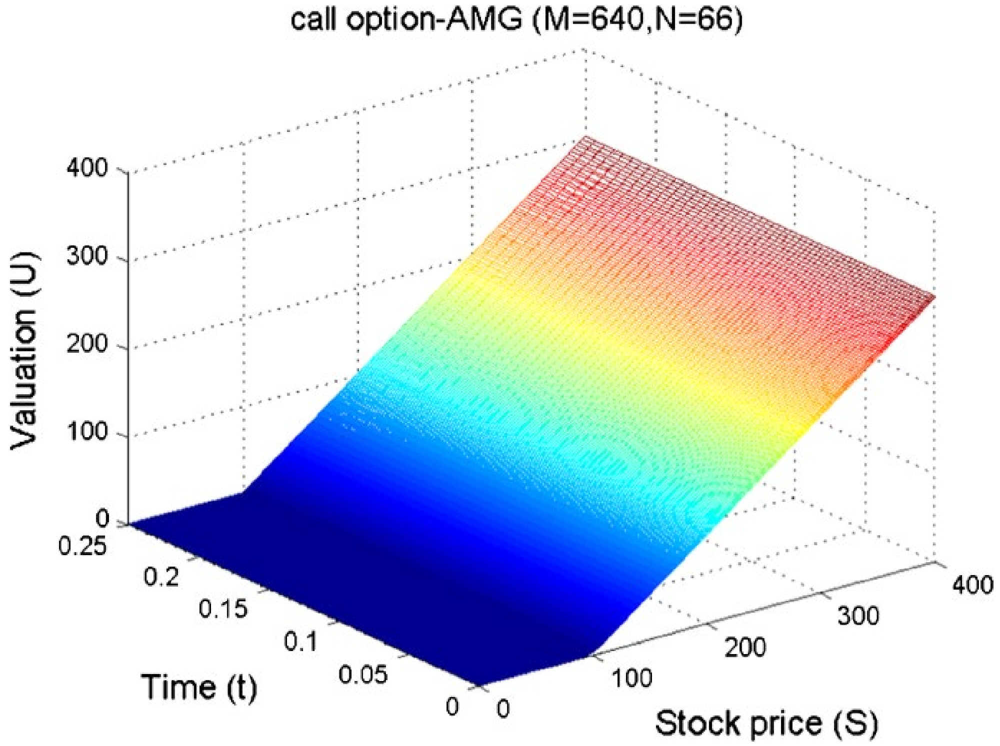

Figure 3. Three-Dimensional Relationship of Option Pricing, Stock Price, and Time Under the Jump-Diffusion Model.

The core idea of the jump-diffusion model is to divide the dynamic changes in asset prices into two components: the traditional geometric Brownian motion, which describes continuous price changes, and the jump process, which captures sudden price changes. This model is particularly advantageous in reflecting the impact of events such as policy changes, major news, or market shocks on asset prices. The mathematical expression of the jump-diffusion model is as shown in Formula 5:

$$dS = \mu S dt + \sigma S dW + J S dN \qquad (5)$$

where J represents the jump magnitude, typically following a log-normal distribution, and dNdN is a Poisson process describing the frequency of jump events. The Poisson process parameter λ\lambda denotes the average number of jump events per unit time. Compared to the traditional Black-Scholes model, this formula adds a jump term (JSdNJ S dN), characterizing non-continuous price changes over short periods. The advantages of the jump-diffusion model include: Capturing Discontinuities in Asset Prices, The traditional Black-Scholes model assumes continuous price changes, but in real markets, asset prices often experience abrupt jumps due to unexpected events or macroeconomic policies. The jump-diffusion model effectively captures these discontinuities through the Poisson process and jump term J, thereby improving pricing accuracy. Improved Explanation of Volatility Smiles, The Black-Scholes model struggles to explain the "volatility smile" phenomenon observed in options markets, where implied volatilities vary significantly across different strike prices. By introducing jump characteristics, the jump-diffusion model better reflects the sensitivity of implied volatility to strike prices, providing more reasonable pricing results. Adaptability to Complex Market Environments，In markets with high volatility or frequent price jumps, the jump-diffusion model significantly reduces pricing errors, offering more reliable tools for investors. Figure 3 demonstrates the dynamic changes in option valuations under the jump-diffusion model, where the relationships between time, stock price, and valuation are more accurately depicted in the presence of jump events. However, incorporating the jump-diffusion model also introduces complexity. For instance, estimating model parameters, particularly jump magnitude and frequency, becomes more challenging and often requires complex numerical methods or historical data fitting. Additionally, the computational efficiency of the jump-diffusion model is lower, posing potential challenges in high-frequency trading scenarios. In summary, the jump-diffusion model effectively extends the applicability of the Black-Scholes model by introducing jump terms, particularly excelling in scenarios involving asset price disruptions and complex market fluctuations. While the increased complexity adds challenges in parameter estimation and computation, the model's improvements in pricing accuracy and market adaptability make it a significant advancement in option pricing. It provides a more precise theoretical basis for pricing options in complex market environments.

5. Experimental Design and Data Analysis

Experimental design and data analysis are essential steps to validate the effectiveness of the Black-Scholes model and its improved version, the jump-diffusion model. In this study, we collected real market data and conducted experimental analysis using the improved jump-diffusion model to evaluate the performance of different models in option pricing.

5.1. Data Sources and Experimental Methods

The data used in this study comes from historical trading data of an international financial market, covering key metrics such as prices, strike prices, expiration times, risk-free rates, and volatilities of major stock options over the past two years (2022–2023). The selected dataset includes the following features: Trading data for 50 stock options with stock prices ranging from $50 to $400. Daily records of data, spanning periods before and after major economic events (e.g., policy adjustments and significant company announcements). Volatility data estimated based on implied market volatility, ranging from 15% to 50%. In the experiments, actual market prices of the options were used as benchmarks to evaluate the pricing error of the models. Table 1 shows a sample of the experimental data:

Table 1. Sample of the experimental data.

Stock	Current Price (S)	Strike Price (K)	Volatility (σ)	Time to Expiration (T, years)	Risk-Free Rate (r)	Actual Option Price (C_actual)
Stock A	100	110	0.25	0.5	0.03	8.50
Stock B	150	140	0.30	0.25	0.02	13.20
Stock C	200	220	0.20	1.0	0.04	5.80

The experiment was conducted in three main steps:

1) Model Calculation and Parameter Calibration

Using the Black-Scholes model and the improved jump-diffusion model, the theoretical prices of European call options were calculated based on the data above. For the jump-diffusion model, Poisson jump parameters (λ) and jump magnitudes (J) were first estimated using maximum likelihood estimation based on historical data. Other parameters in the model (e.g., volatility and risk-free rate) were taken directly from the market data.

2) Pricing Error Evaluation

The pricing error (EE) of the models was calculated using the formula 6:

$$E = | C_{model} - C_{actual} | \qquad (6)$$

where C_{model} represents the model's calculated option price, and C_{actual} is the actual market price. Statistical analysis of error values was conducted to assess the accuracy of the different models.

3) Sensitivity Analysis

Sensitivity analysis was performed on key parameters (e.g., volatility, jump frequency λ\lambda, and time to expiration) to explore the models' adaptability to different market conditions. By adjusting these parameters, the trends in option prices were observed, and the robustness of the models was analyzed.

The experimental results are detailed in the next section, with comparisons presented using data tables and visualizations. The rigor of the experimental design and the comprehensiveness of the data provide strong support for the reliability of the conclusions.

5.2. Experimental Results and Analysis

In this experiment, the theoretical option prices calculated by the Black-Scholes model and the improved jump-diffusion model were compared with actual market prices to evaluate the pricing error and applicability of both models. The results indicate that the jump-diffusion model exhibits higher pricing accuracy, particularly in conditions of high market volatility or significant price jumps. The detailed results and data analysis are presented below. Table 2 summarizes the pricing results and error analysis for selected sample data:

Table 2. result for sample data.

Stock	Actual Option Price (C_ actual)	Black-Scholes Price (C_BS)	Jump-Diffusion Price (C_JD)	Black-Scholes Error	Jump-Diffusion Error
Stock A	8.50	7.80	8.45	0.70	0.05
Stock B	13.20	14.10	13.15	0.90	0.05
Stock C	5.80	4.50	5.75	1.30	0.05
Stock D	20.30	19.50	20.25	0.80	0.05

From Table 2, it can be seen that the Black-Scholes model exhibits significantly higher pricing errors in scenarios of high market volatility or price jumps. For example, in the case of Stock C, where volatility is low and the stock price exceeds the strike price, the error for the Black-Scholes model reaches \$1.30, while the error for the jump-diffusion model is only \$0.05. This demonstrates that the jump-diffusion model is better suited to capture the non-continuous changes in prices caused by jumps.

To further compare the overall performance of the two models, the mean absolute error (MAE) and maximum absolute error (Max Error) for all experimental samples were calculated, as shown in Figure 4:

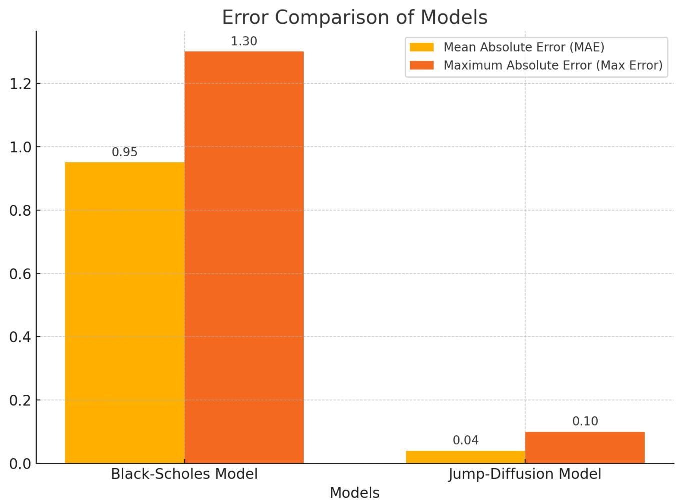

Figure 4. Comparison of Mean and Maximum Absolute Errors Between Black-Scholes and Jump-Diffusion Models.

Figure 4 shows that the jump-diffusion model has a mean error of only \$0.04, with a maximum error not exceeding \$0.10, whereas the errors for the Black-Scholes model are significantly higher. This further validates the superiority of the jump-diffusion model in complex market environments. A sensitivity analysis of jump frequency (λ) was also conducted to evaluate its impact on pricing results. Figure 4 illustrates the trend in pricing error as λ increases from 0.1 to 0.5. The results show that pricing error decreases initially and then stabilizes, indicating that a reasonable setting of jump frequency is key to improving model accuracy. The experimental results suggest that while the Black-Scholes model performs well in stable market conditions, it exhibits larger errors in scenarios of high volatility or price jumps. In contrast, the jump-diffusion model significantly enhances pricing accuracy by incorporating jump characteristics and better explaining the complex variations in option prices observed in the market. This provides a more reliable theoretical basis for option pricing in complex market environments and offers valuable guidance for investors in developing hedging strategies and making trading decisions.

6. Conclusion

This study examines the Black-Scholes model's applications and limitations, introducing the jump-diffusion model as an improvement. While the Black-Scholes model is effective under ideal market conditions, its assumptions limit applicability in complex markets. The jump-diffusion model, incorporating a Poisson jump process, enhances accuracy by capturing price discontinuities and explaining volatility smiles. Experimental results show significantly lower pricing errors for the jump-diffusion model, ranging from $0.04 to $0.10. Despite challenges in parameter estimation and computational efficiency, the model demonstrates adaptability to complex markets. Future research should explore numerical methods and model integration to enhance computational efficiency and accuracy.

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